trix elements. Thus the errors are of order unity rather than of order 30 000 which should not be too important since only relative strengths and positions of structure in the enhancement factors were of interest. Since the models upon which calculations were performed were sufficiently crude, the resulting numerical discrepancy should not be regarded as physically meaningful. The calculations were purely model calculations to illustrate physical points, and the importance of the numerical agreement between theory and experiment was simply to show that experimental field-emission and resonance tunneling data could be analyzed in a reasonably straightforward manner by choosing realistic parameters characterizing the interacting atom-metal system.

It was the hope that the simple model calculations would point out a new physical effect and stimulate further more realistic calculations. In Ref. 2, the fact that d-band tunneling is significantly lower than s-band tunneling was illustrated. As a result of this observation, Politzer and Cutler[4] have since confirmed this point, improved the model, and calculated numbers which are in qualitative accord with those obtained in Ref. 2.

The key spectroscopic results in the resonance tunneling analysis,[3] namely, that the ns level widths 2Δ are about 1 eV and the $(n-1)d$ level widths are about 0.1 eV in adsorbed alkaline-earth atoms, remain unaffected by Glasser's comments. The tunneling theory presented in Ref. 3, in which it is shown how experimental results can be analyzed in terms of level-width parameters, is in accord with other resonance tunneling theories.[5] Thus if we regard $2\Delta = \Gamma$ as a fitting parameter and do a fit to the data of Plummer and Young[6] with the tunneling theory result, Eqs. (13) and (19) of Ref. 3, then similar values for Δ as calculated[3] will obtain.

In conclusion, although the methods of obtaining the matrix elements in Refs. 2 and 3 must be revised as indicated by Glasser,[1] when this is done it is felt that the altered results will not be sufficiently different from the reported ones to make it worth doing.

[1]M. L. Glasser, preceding paper, Phys. Rev. B 3, 1772 (1971).
[2]J. W. Gadzuk, Phys. Rev. 182, 416 (1969).
[3]J. W. Gadzuk, Phys. Rev. B 1, 2110 (1970).
[4]B. A. Politzer and P. H. Cutler, Surface Sci. 22, 277 (1970).
[5]J. N. L. Connor, Phys. Rev. B (to be published).
[6]E. W. Plummer and R. D. Young, Phys. Rev. B 1, 2088 (1970).

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Comments on the Upper Critical Field of Type-II Two-Band Superconductors

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The justification for the existence of interband coupling between the pair wave functions Δ_s^* and Δ_d^* for two-band superconductors in the dirty limit is explained in greater detail.

In two previous papers[1,2] by the present author, a model two-band superconductor in the dirty limit was studied. It is found that even without interband BCS coupling, the pair wave functions $\Delta_s^*(\vec{x})$ and $\Delta_d^*(\vec{x})$ (for the s band and the d band, respectively) can be coupled through vertex corrections due to interband impurity scattering. As a result, we find that in the temperature region between the s-band transition temperature and the d-band transition temperature, the upper critical field is uniformly lowered by the interband impurity scattering (see I) and in the temperature region just above zero, $T \gtrsim 0$, the upper critical field for a superconductor with both bands in the superconducting phase is higher than that for a superconductor with d band in the superconducting phase and s band in the normal phase (see II). In a recent paper by Sung,[3] it is asserted that the calculation performed in I is not correct, and his reason is that he cannot think of any justification of Eqs. (I34) and (I35), which couple the pair wave functions $\Delta_s^*(\vec{x})$ and $\Delta_d^*(\vec{x})$ through interband impurity scattering. As a matter of fact, his assertion is not correct. A justification in the spirit of the Feynman-diagram technique for obtaining vertex corrections[4] is given in I. For the convenience of the readers of these papers, in this note we shall explain this in greater detail.

We start with Eqs. (I31) and (I32),

$$\Delta_s^*(\vec{x}) = g_s T \sum_\nu \int d^3l \, \langle G_{0ss}^A(\vec{l},\vec{x};-z_\nu) \times G_{0ss}^A(\vec{l},\vec{x};z_\nu)\, \Delta_s^*(\vec{l})\rangle_{\text{vert.corr.}}$$

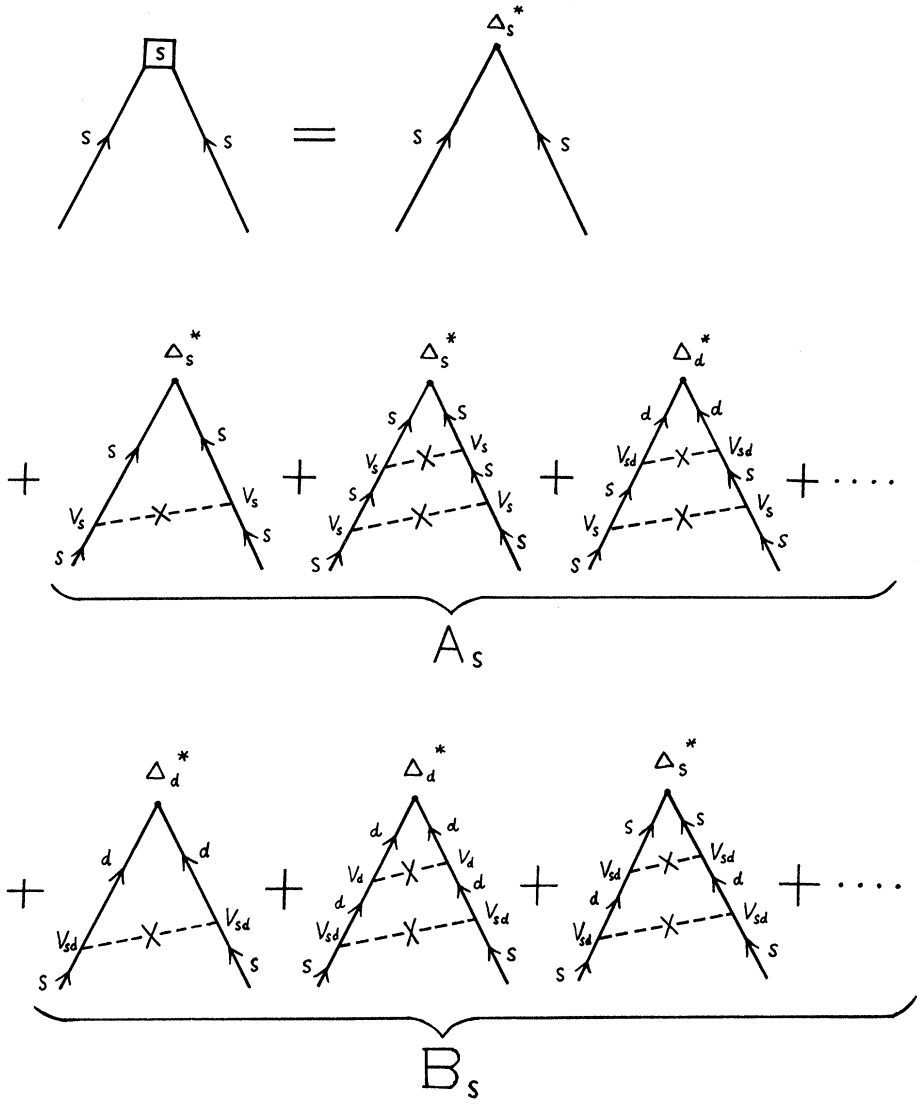

FIG. 1. Diagram expansion for the s-vertex function. A similar expansion can be plotted for the d-vertex function.

and

$$\Delta_d^*(\vec{x}) = g_d T \sum_\nu \int d^3l \, \langle G^A_{0dd}(\vec{l}, \vec{x}; -z_\nu) \times G^A_{0dd}(\vec{l}, \vec{x}; z_\nu) \, \Delta_d^*(\vec{l}) \rangle_{\text{vert.corr.}} .$$

Instead of considering directly the integrands given above, we consider their Fourier-transformed quantities, the vertex functions,

$$K_{ss}(\vec{p}_1, \vec{p}_2; z_\nu) = \langle G^A_{0ss}(\vec{p}_1, -z_\nu) \times G^A_{0ss}(\vec{p}_2, z_\nu) \, \Delta_s^*(\vec{p}_1 + \vec{p}_2) \rangle_{\text{vert.corr.}}$$

and

$$K_{dd}(\vec{p}_1, \vec{p}_2; z_\nu) = \langle G^A_{0dd}(\vec{p}_1, -z_\nu) \times G^A_{0dd}(\vec{p}_2, z_\nu) \, \Delta_d^*(\vec{p}_1 + \vec{p}_2) \rangle_{\text{vert.corr.}} .$$

Each vertex function can be represented by a "square" with two lines representing two incoming Green's functions attached. The "square" then represents the sum of all diagrams due to the intraband as well as the interband impurity scattering, which are consistent with the two incoming Green's functions. In Fig. 1, the s-vertex function is expanded in terms of its consistent diagrams, the s diagrams. Similarly, we can expand the d-vertex function in terms of the d diagrams. We notice that an s diagram (or a d diagram) can end up either with $\Delta_s^*(\vec{p}_1 + \vec{p}_2)$ or with $\Delta_d^*(\vec{p}_1 + \vec{p}_2)$. This is physically acceptable, because both the pair wave functions Δ_s^* and Δ_d^* can exist simultaneously as molecular fields in the two-band system, when both bands are in the superconducting phase. Further, the s

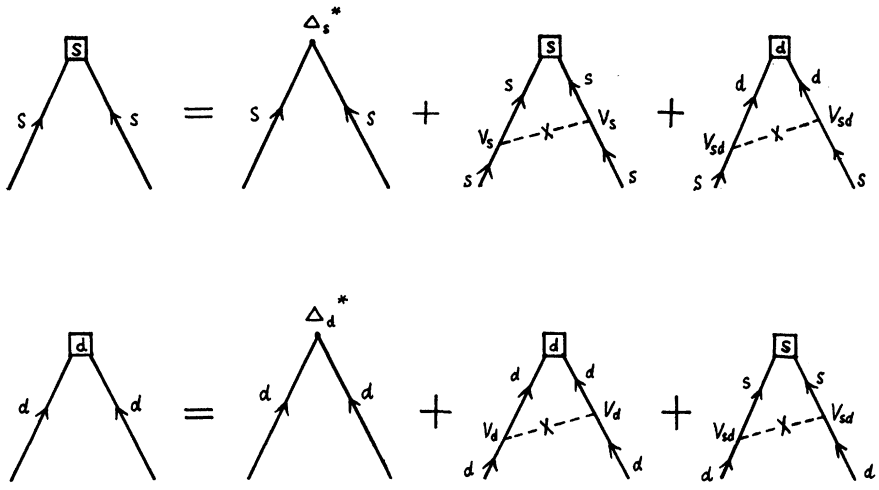

FIG. 2. Diagram equations obtained from Fig. 1. These equations are used to obtain Eqs. (I34) and (I35).

diagrams, other than the lowest-order one which does not involve impurity scattering, can be classified into two groups; group A_s consisting of those diagrams starting with intraband impurity scattering, and group B_s consisting of those diagrams starting with interband impurity scattering. With this, we immediately reach the diagram equations given in Fig. 2, which in turn lead us to our basic mathematical equations, Eqs. (I34) and (I35). Physically, this is also justifiable, as the impurities can scatter a pair of electrons in the s band into the d band, and vice versa.

In this connection, the "Born-approximation" argument given by Sung is misleading (see p. 552 of S), and only serves his purpose of avoiding the inclusion of the groups of diagrams, B_s and B_d, to appear in his calculation. As a matter of fact, the inclusion of the diagrams B_s and B_d into his low-impurity-density theory is possible. And undoubtedly, this would further complicate his already complicated Eqs. (S28) and (S29).

Finally, it is enlightening to notice that for two-band superconductors in the pure limit, the pair wave functions $\Delta_s^*(\vec{x})$ and $\Delta_d^*(\vec{x})$ are intrinsically coupled via a parameter, J^2/J_sJ_d [here, $J_{s(d)}$ is the intraband BCS coupling constant for the s (d) band, and J the interband BCS coupling constant], which is estimated by Sung to be very small, $\sim 10^{-3}$ (see S). Therefore, it is expected that our dirty-limit theory (I and II) can be valid for two-band superconductors with impurity-density sufficiently high such that the coupling due to the impurity-scattering vertex corrections becomes more important than the intrinsic coupling, J^2/J_sJ_d, which is very well discussed by Sung for superconductors in the pure limit. The question, under what impurity density the dirty-limit theory starts to be more adequate to treat a two-band superconductor, would eventually be clarified when more experimental information is available in future.

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[1]W. S. Chow, Phys. Rev. 176, 525 (1968), hereafter referred to as I.

[2]W. S. Chow, Phys. Rev. 179, 444 (1969), hereafter referred to as II.

[3]C. C. Sung, Phys. Rev. 187, 548 (1969), hereafter referred to as S.

[4]See for example, F. Mandl, *Introduction to Quantum Field Theory* (Interscience, New York, 1958), p. 145; A. A. Abrikosov, L. P. Gorkov, and I. F. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Englewood Cliffs, N. J., 1963), p. 332.